Calculation Policy

Introduction

At Greenleas we believe that in order for children to fully understand how to solve a calculation, they need to fully understand the fundamental principles behind the calculation. In order to develop understanding, children need to be immersed in mathematics through: concrete experiences and manipulatives; images and visual representations. To achieve this, we follow a mastery approach to mathematics learning and teaching which integrates mathematical thinking; rich discussion and verbal reasoning.

Our calculation policy is therefore split into the following areas:

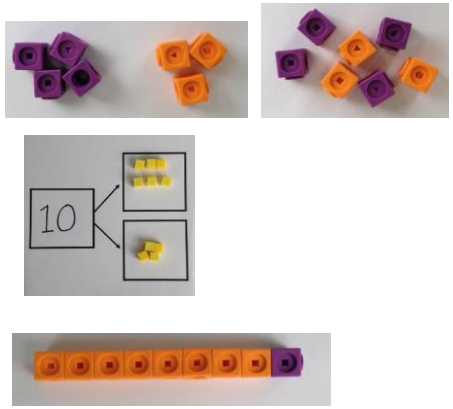
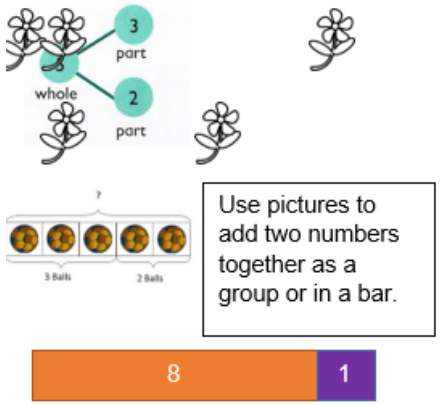
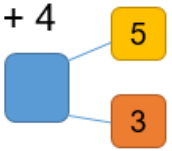
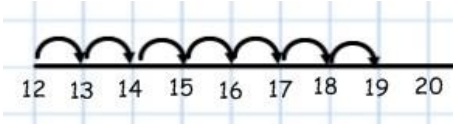
Concrete	Pictorial	Abstract
Concepts are represented physically using objects and manipulatives- supporting children in their understanding and reasoning.	Calculations are represented visually, either as jottings or as a visual model. Pictorial representations can work as a way to solve a calculation or as a way to help children explain how a formal method works.	Mathematical thinking is shown through symbols, often as a formal written method. Children should have a deep understanding of the fundamental principles behind a calculation and be able to explain methods in other ways.
Reasoning		
Children verbally explain what they are doing when solving a calculation. Talking is an essential part of learning and allows children to consolidate, embed and deepen their understanding of how to solve a calculation.		

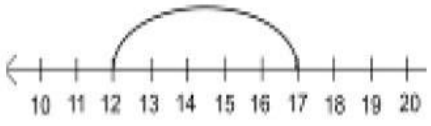
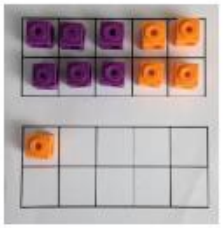
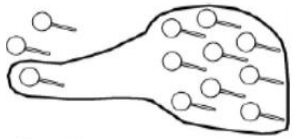
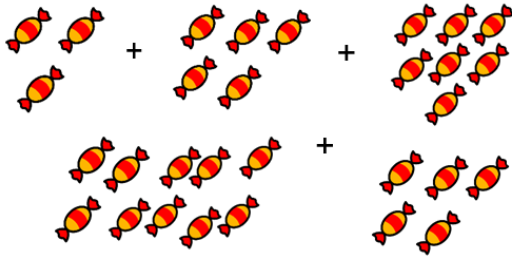
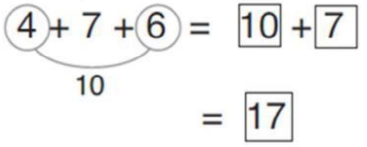
Addition

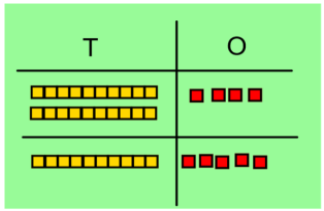
EYFS
Children need to recognise a small group of objects/dots/fingers as the total without counting them. They use a range of concrete resources before moving onto recording and can recognise amounts without counting.

Key Stage One
Children are taught to add using concrete resources in order to identify and represent the different parts of a calculation. They add one-digit and two-digit numbers to 20, including zero. Much time is spent securing the place value of tens and ones. Children master the concept of exchanging, with ten 'ones' being exchanged for one 'ten' where the ones digit goes over ten, leading to the introduction on columnar addition in Year 2.

Key Stage Two
The fundamentals of columnar addition are established by the end of Year 2. This method is then consolidated from Year 3 onwards in order for children to become efficient with adding increasingly larger numbers. Children should be fluent with the recall of single-digit addition facts, allowing the individual calculations in each step to be quickly solved. In Year 4 children are introduced to decimals and connect this concept with their understanding of place value.

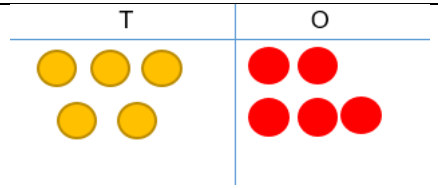
	Concept	Concrete	Pictorial	Abstract
EYFS Year 1	**Combining two parts to make a whole: part-whole model**			4 + 3 = 7 10 = 6 + 4  Use the part-part whole diagram as shown above to move into the abstract. Four is a part, three is a part, and the whole is Seven.
EYFS	**Starting at the bigger number and counting on**	 Start with the larger number on the	12 + 7 = 19 	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer. 'Counting' becomes

Year 1		bead string and then count on the smaller number 1 by 1 to find the answer.	Start at the larger number on the number line and count on in ones or in one jump to find the answer.  	automatic from applying fluency knowledge rather than counting in ones.
Year 1	**Regrouping to make 10**	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. **3 + 9 =** 9 + 5 = 14 1 4 +1 +4 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	9 + 5 = 14 If I am at nine, how many more do I need to make 10? How many more do I need to add on now?
Year 2	**Adding three single digits**	4 + 7 + 6 = 17  Put 4 and 6 together to make 10. Add on 7. Make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 (if possible), and then add on the remaining number.

<table>
<tr>
<td>Year 2</td>
<td>Column method with no exchange</td>
<td>


24 + 15 =

Add the ones together first, then add the tens.

Use Base 10 blocks first before moving onto place value arrow cards/ counters.
</td>
<td>


After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.
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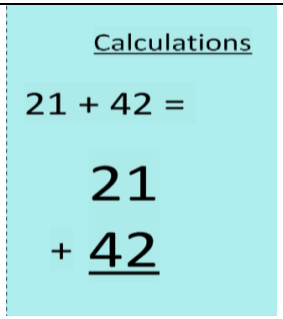
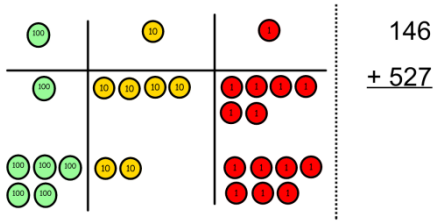
<td>


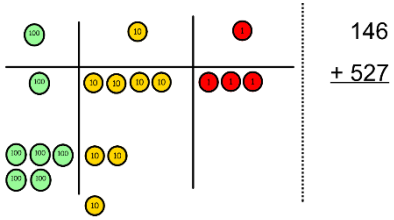
Children link this method with partitioning, working with values of ones and tens.
</td>
</tr>
<tr>
<td>Year 2

KS2</td>
<td>Column method with exchange</td>
<td>
Begin the concrete stage with Base 10, then move onto using Place Value counters.

Make both numbers on a place value grid.

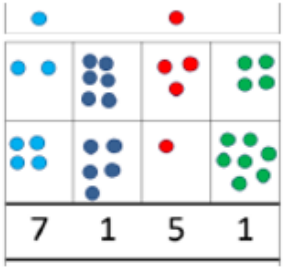


Begin by adding the ones together, exchange 10 ‘ones’ for 1 ‘ten’.



Continue to add each column, exchanging 10 counters from one column for the next as needed.
</td>
<td>
Children can draw a pictorial representation of the columns and concrete resources to further support their learning and understanding.

The language used needs to refer to the place value column …
4 ones and 7 ones equals eleven ones.
3 tens add 1 ten. plus the extra one ten.



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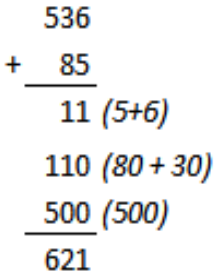
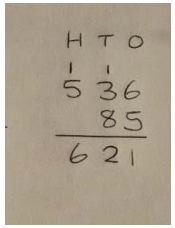
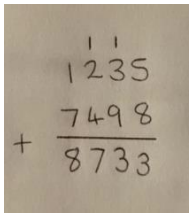
<td>


Expanded method bridges understanding between the concrete and the abstract





It is important that children refer to the place value of each column when adding and maintain an understanding of the overall calculation. They may find Place Value Headings useful.
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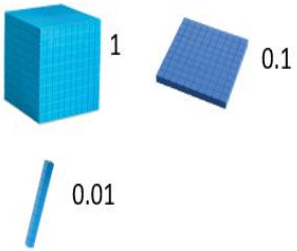
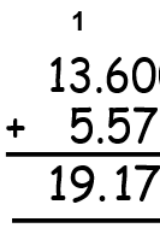
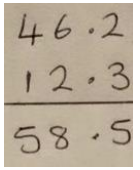
KS2	**Column method with decimals**	Using Base 10 resources, the place value and relative size of each decimal place can be visualised  .	 Pictorial representations help children to reason about calculations. The bar model clearly shows that the answer will be less than 20.	Make links with money when introducing decimals.   Zero is used as a place holder.
	Vocabulary	Key Language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to', 'is the same as'.		

Subtraction

EYFS

In practical contexts, children use concrete objects to show how objects can be taken away.

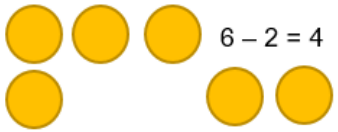
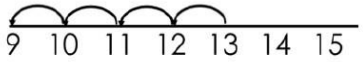
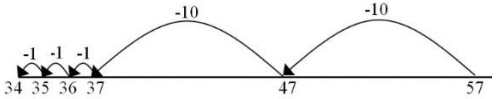
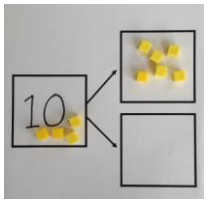
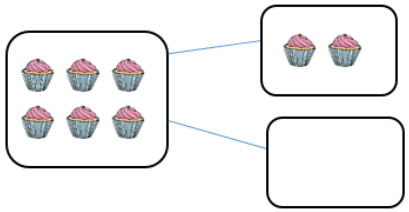
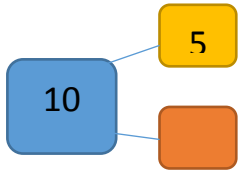
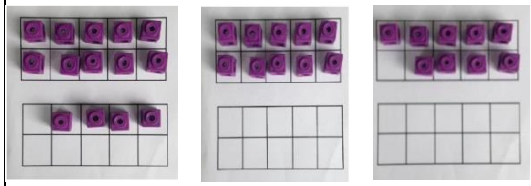
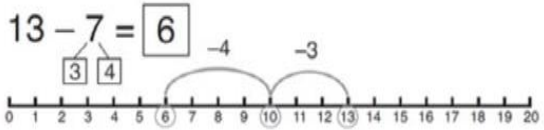
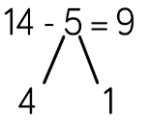
Key Stage One

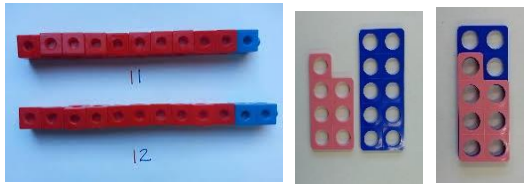
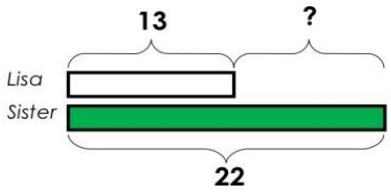
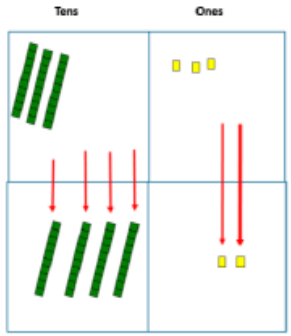
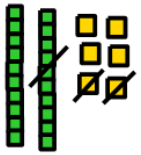
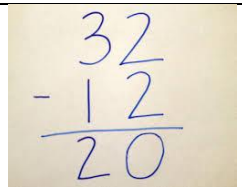
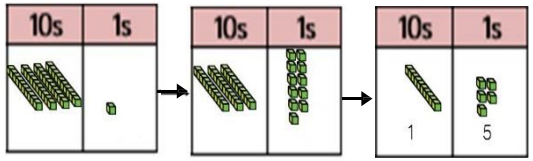
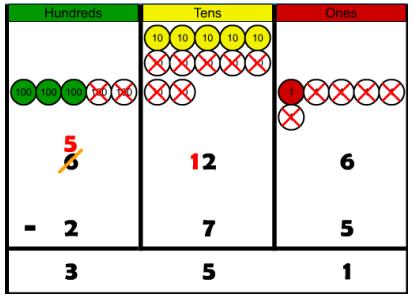
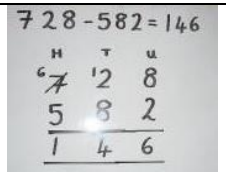
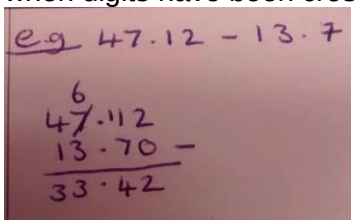
Children are taught that subtraction is the inverse to addition. They use concrete resources to compare numbers, find the difference, or physically remove a number of objects. When using pictorial jottings children can show subtraction by counting back on a number line or by 'crossing through' the subtracted amount. They also use their knowledge of place value to exchange one 'ten' for ten 'ones' when subtracting two-digit numbers.

Key Stage Two

As with addition, the fundamentals of columnar subtraction are in place by the end of Year 2. From Year 3 onwards, children consolidate their understanding of this method and become confident in subtracting increasingly larger numbers. Children are also expected to have developed fluency with subtraction facts within 20 so that they efficiently process each calculation. They are introduced to decimals in Year 4 and this is worked into columnar subtraction through their secure understanding of place value.

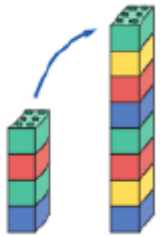
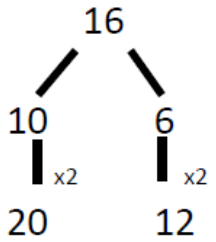
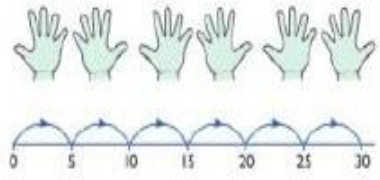
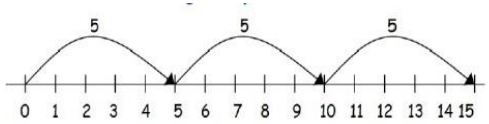
	Concept	Concrete	Pictorial	Abstract

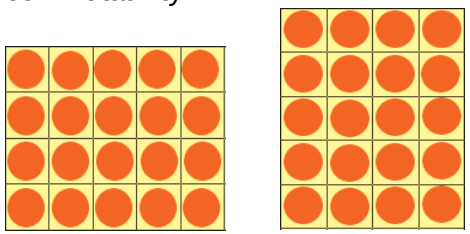
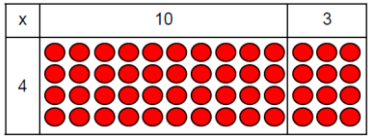
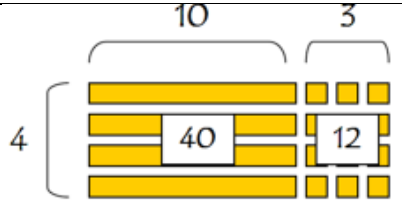
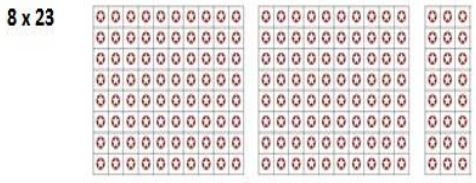
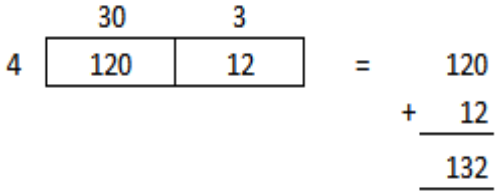
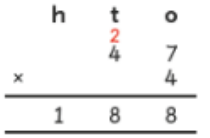
EYFS	**Taking away ones**	In a practical context, use physical objects, counters, cubes etc. to show how objects can be taken away. 	Cross out drawn objects to show what has been taken away. 	8 – 2 = 6
EYFS Year 1	**Counting back**	Move the beads along your bead string as you count backwards in ones.  13 – 4 = Use counters and move them away from the group as you take them away counting backwards as you go.	Count back on a number line.  Start at the bigger number and count back, becoming more efficient at using number facts rather than counting in ones.  Counting back using two 2-digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help initially; then knowledge and fluency of number facts. 13 – 4 =
EYFS Year 1	**Part Part Whole Model**	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part?	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model. 10 – 5 = 5
Year 1 Year 2	**Make 10**	14 – 5 =  Make 14 on the ten frame. Take away four first to make 10 and then take away one more. You have taken away 5.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether.	14 – 5 = How many do we take off to reach the next 10? How many do we have left to take off? 

Year 1 Year 2	**Finding the difference**	Compare amounts and objects to find the difference. 	Comparison Bar Models *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches. Sam has 15 sandwiches. Find the difference between the number of sandwiches.
Year 2 KS2	**Column method with no exchange**	 Use Base 10 to make the bigger number. Show how partitioning is used to subtract, then take away.	Draw the Base 10 or place value counters alongside the written calculation to help to show working. 26 - 12 = 14 	 32 – 12 = 20
Year 2 KS2	**Column method with exchange**	Use Base 10 resources before moving onto Place Value counters. Start with one exchange before moving on to subtractions with two exchanges.  41 – 26 = Children must understand that they still have 41 because 41 = 30 + 11.	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. 	 Show children how the concrete links to the written method so they understand what has happened when digits have been crossed out.  This will lead to understanding of subtracting any number.
	Vocabulary	Key language:take away,less than,the difference,subtract,minus,fewer,decrease.		

Multiplication

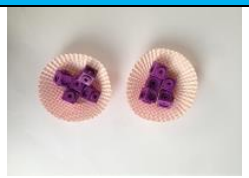
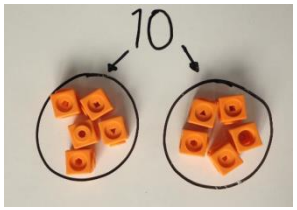
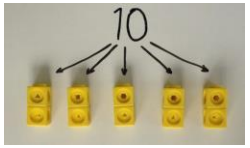
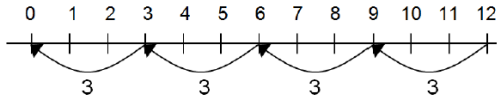
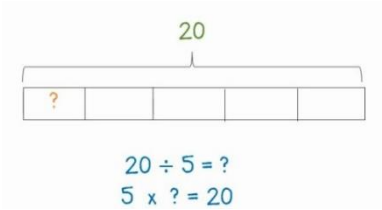
EYFS
Children use concrete objects to double numbers.
Key Stage One
In Key Stage 1 children are taught to count in multiples of 2, 5 and 10, using objects and images to represent this. They understand multiplication as repeated addition and that the groups must be equal. Working practically with arrays demonstrates that multiplication is commutative.
Key Stage Two
By the end of Year 4 children should be fluent in all multiplication tables facts up to 12 x 12 which will allow them to make links and quickly solve the individual steps within written calculations.

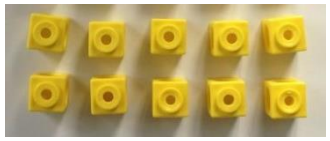
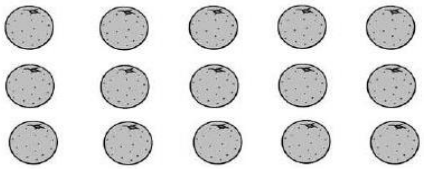
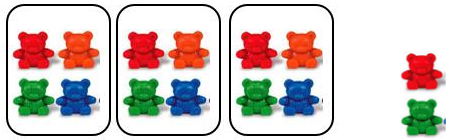
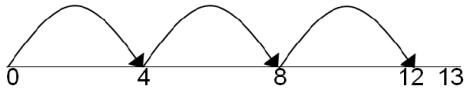
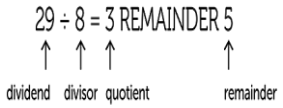
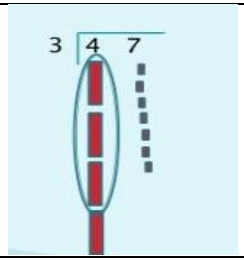
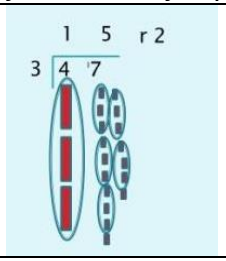
	Concept	Concrete	Pictorial	Abstract
EYFS / Y1 / Y2	**Doubling**	Use practical activities to show how to double a number. 	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together.
Year 1	**Counting in Multiples**	Count in multiples supported by concrete objects in equal groups. Ensure children do not count in ones. 	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Year 2	**Repeated addition of equal groups**	Use different objects to recognise and add equal groups. 	5 + 5 + 5 = 15 	Write addition sentences to describe objects and pictures. 

Year 2	**Arrays showing commutative multiplication**	Create arrays to show multiplication sentences. .  2 x 6 = 12	Use arrays in different orientations to show commutativity.  4 x 5 + 20 5 x 4 + 20	Use arrays to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 x 5 + 15
Year 3	**Partition to multiply**	Show the link with arrays to introduce the grid method 4 x 10 = 4 x 3 =  Move onto Base 10, and then Place Value Counters, towards a more compact method.	 40 + 12 = 52 4 x 13 = 52	13 x 4 12 40 52 4 x 13 = 52
KS2	**Column Multiplication**	 8 x 23 = 8 x 10 + 8 x 10 + 8 x 3	Children draw pictorial representations, using place value counters. 33 x 4 = 	Expanded method leading to the compact method. 33 x 4 12 (4 x 3) 120 (4 x 30) 132 
	Vocabulary	Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.		

Division

EYFS
Children share using concrete objects.
Key Stage One
Initially children understand division as sharing. They also learn to halve numbers. As children begin to deal with larger numbers, they realise sharing is not an efficient way of dividing and use their knowledge of counting in multiples of 2, 5 and 10 to solve problems involving division.
Key Stage Two
Children are taught to link division to known multiplication facts. It is therefore essential that times tables facts up to 12 x 12 can be derived.

	Concept	Concrete	Pictorial	Abstract
EYFS / Year 1	**Sharing into equal groups**	   I have 10 cubes. Can you share them equally into two groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Only introduce symbols when appropriate. Share 9 buns between three people. 9 ÷ 3 = 3
Year 1 / Year 2	**Division as grouping**	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  How many groups of 2 are in 10?  How many groups of 5 are in 35?	The number of jumps equals the number of groups.  	40 ÷ 5 = 8 Divide 40 into groups of 5. How many groups are there? 5 10 15 20 25 30 35 40 40 ÷ 5 = 8 2 4 6 8 10 10 ÷ 2 = 5

Year 2	**Division within arrays**	 10 ÷ 5 = 2 2 x 5 = 10 Link division to multiplication by creating an array and thinking about the number sentences that can be created.	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 5 = 35 5 x 7 = 35 35 ÷ 7 = 5 35 ÷ 5 = 7
Year 2 Year 3	**Division with a remainder**	14 ÷ 4 = Divide objects into equal groups and see how many are remaining. 	 Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.	Complete written divisions and show the remainder using 'r'. 
KS2	**Short Division**	 47 ÷ 3 = Begin with Base 10, then use Place Value counters.	 47 ÷ 3 = 15 r 2 How many groups of 3 are there in 47? How many groups of 3 tens, 3 ones?	98 ÷ 7 becomes 1 4 7 9 8 Answer: 14 432 ÷ 5 becomes 8 6 r 2 5 4 3 2 Answer: 86 remainder 2
	Vocabulary	Key language: share, group, divide, divided by, half.		

Progression in Difficulty in Calculations

Addition	**Subtraction**
No exchange	No exchange
Extra digit in the answer	Fewer digits in the answer
Exchanging ones to tens	Exchanging tens for ones
Exchanging tens to hundreds	Exchanging hundreds for tens
Exchanging ones to tens and tens to hundreds	Exchanging hundreds to tens and tens to ones
More than two numbers in calculation	As 5 but with different number of digits
As 6 but with different number of digits	Decimals up to 2 decimal places (same number of decimal places)
Decimals up to 2 decimal places (same number of decimal places)	Subtract two or more decimals with a range of decimal places
Add two or more decimals with a range of decimal places	
Short Multiplication	**Short Division**
TO x O no exchange	TO ÷ O no exchange no remainder
TO x O extra digit in the answer	TO ÷ O no exchange with remainder
TO x O with exchange of ones into tens	TO ÷ O with exchange no remainder
HTO x O no exchange	TO ÷ O with exchange, with remainder
HTO x O with exchange of ones into tens	Zero in the quotient e.g. 816 ÷ 4 = 204
HTO x O with exchange of tens into hundreds	As 1-5 HTO ÷ O
HTO x O with exchange of ones into tens and tens into hundreds	As 1-5 greater number of digits ÷ O
As 4-7 but with greater number digits x O	As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3
O.t x O no exchange	Where the divisor is a two-digit number Remainders
O.t with exchange of tenths to ones	Whole number remainder
As 9 - 10 but with greater number of digits which may include a range of decimal places x O	Remainder expressed as a fraction of the divisor
	Remainder expressed as a simplified fraction
	Remainder expressed as a decimal